

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
International General Certificate of Secondary Education

**MATHEMATICS**

**0580/04**  
**0581/04**

Paper 4 (Extended)

May/June 2006

Additional Materials: Answer Booklet/Paper  
Electronic calculator  
Geometrical instruments  
Graph paper (2 sheets)  
Mathematical tables (optional)  
Tracing paper (optional)

**2 hours 30 minutes**

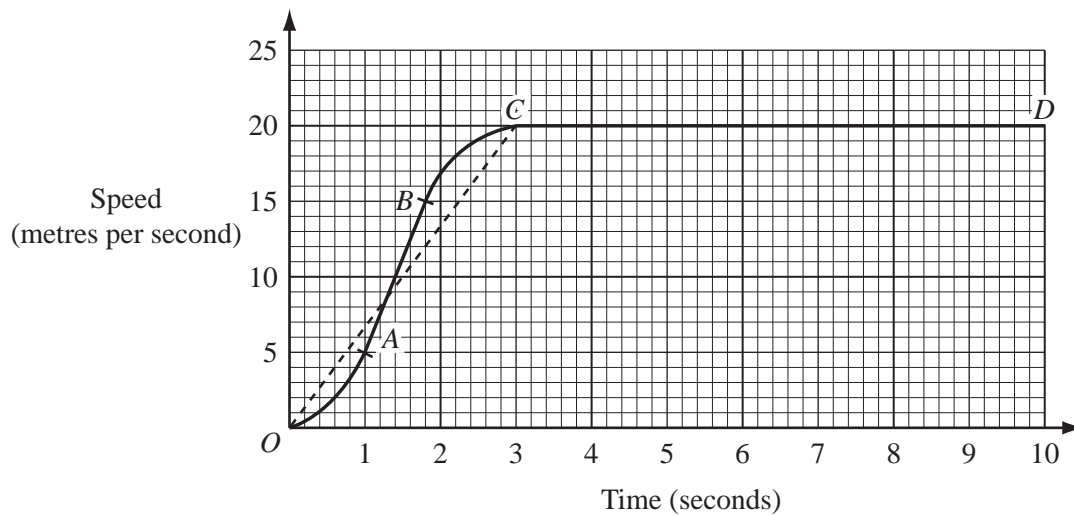
**READ THESE INSTRUCTIONS FIRST**

Write your answers and working on the separate Answer Booklet/Paper provided.  
Write your name, Centre number and candidate number on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.  
At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer.  
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.  
The total of the marks for this paper is 130.  
Electronic calculators should be used.  
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.  
Give answers in degrees to one decimal place.  
For  $\pi$  use either your calculator value or 3.142.

- 1 (a) A train completed a journey of 850 kilometres with an average speed of 80 kilometres per hour. Calculate, giving **exact** answers, the time taken for this journey in
- hours,
  - hours, minutes and seconds. [1]
- (b) Another train took 10 hours 48 minutes to complete the same 850 km journey.
- It departed at 1920.  
At what time, on the next day, did this train complete the journey? [1]
  - Calculate the average speed, in kilometres per hour, for the journey. [2]
- (c)



The **solid** line  $OABCD$  on the grid shows the first 10 seconds of a car journey.

- Describe briefly what happens to the **speed** of the car between  $B$  and  $C$ . [1]
- Describe briefly what happens to the **acceleration** of the car between  $B$  and  $C$ . [1]
- Calculate the acceleration between  $A$  and  $B$ . [2]
- Using the **broken** straight line  $OC$ , estimate the total distance travelled by the car in the whole 10 seconds. [3]
- Explain briefly why, in this case, using the broken line makes the answer to **part (iv)** a good estimate of the distance travelled. [1]
- Calculate the average speed of the car during the 10 seconds.  
Give your answer in kilometres per hour. [2]

2

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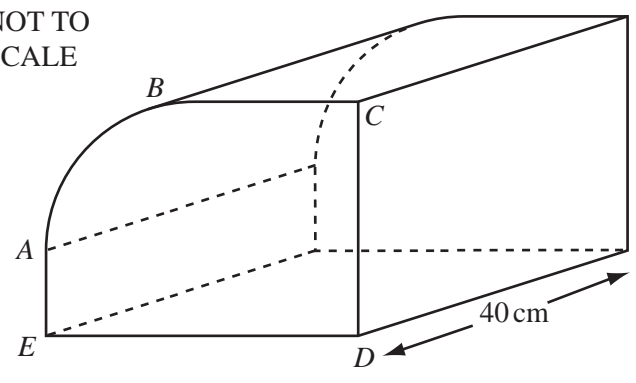


Diagram 1

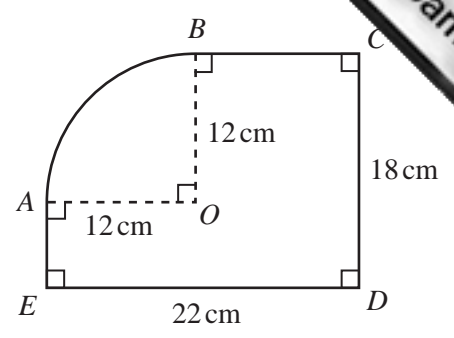


Diagram 2

Diagram 1 shows a closed box. The box is a prism of length 40 cm. The cross-section of the box is shown in Diagram 2, with all the right-angles marked.  $AB$  is an arc of a circle, centre  $O$ , radius 12 cm.  $ED = 22$  cm and  $DC = 18$  cm.

Calculate

- (a) the perimeter of the cross-section, [3]
- (b) the area of the cross-section, [3]
- (c) the volume of the box, [1]
- (d) the **total** surface area of the box. [4]

3 Answer the whole of this question on a sheet of graph paper.

(a) Find the values of  $k$ ,  $m$  and  $n$  in each of the following equations, where  $a > 0$ .

- (i)  $a^0 = k$ , [1]
- (ii)  $a^m = \frac{1}{a}$ , [1]
- (iii)  $a^n = \sqrt{a^3}$ . [1]

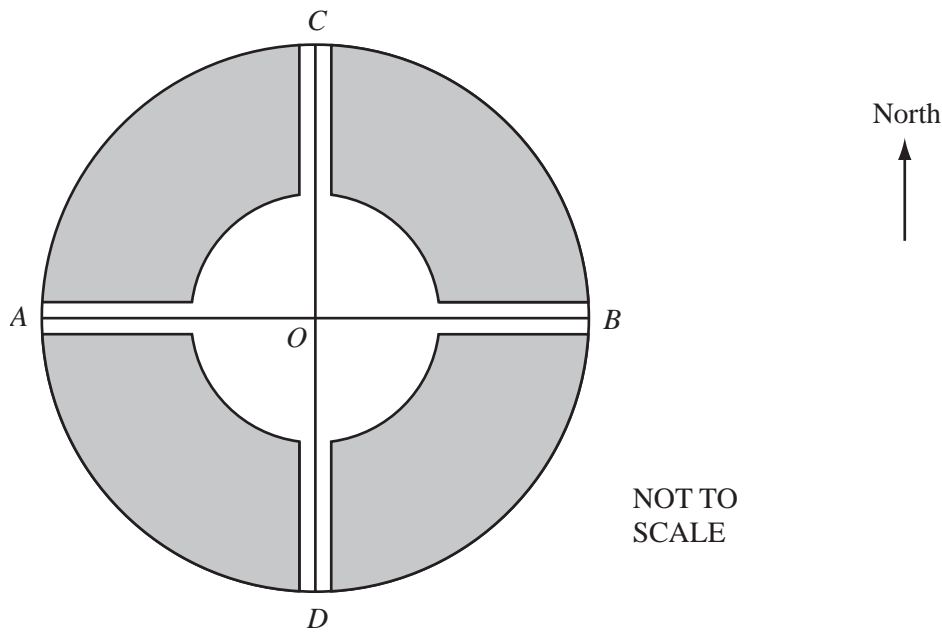
(b) The table shows some values of the function  $f(x) = 2^x$ .

$x$	-2	-1	-0.5	0	0.5	1	1.5	2	3
$f(x)$	$r$	0.5	0.71	$s$	1.41	2	2.83	4	$t$

- (i) Write down the values of  $r$ ,  $s$  and  $t$ . [3]
- (ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an  $x$ -axis from -2 to 3 and a  $y$ -axis from 0 to 10. [1]
- (iii) On your grid, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 3$ . [4]

(c) The function  $g$  is given by  $g(x) = 6 - 2x$ .

- (i) On the same grid as **part (b)**, draw the graph of  $y = g(x)$  for  $-2 \leq x \leq 3$ . [2]
- (ii) Use your graphs to solve the equation  $2^x = 6 - 2x$ . [1]
- (iii) Write down the value of  $x$  for which  $2^x < 6 - 2x$  for  $x \in \{\text{positive integers}\}$ . [1]



The diagram shows a plan for a new city.  
 It is to be built inside a circle of radius 5 km.  
 The areas where homes can be built are shaded on the diagram.  
 The homes must be at least 2 km from the centre of the city,  $O$ .  
 The homes must also be at least 0.5 km from two main roads  $CD$  and  $AB$ , which are in North-South and West-East directions.

- (a) Using 1 cm to represent 1 km, make an **accurate** scale drawing showing the areas for the homes.  
 (You do not need to shade these areas.) [4]
- (b) The town hall,  $T$ , will be built so that it is equidistant from the roads  $OA$  and  $OC$ .  
 It will be 1 km from  $O$  **and** West of  $CD$ .
- (i) On your scale drawing, using a straight edge and compasses only, draw the locus of points, inside the town, which are equidistant from  $OA$  and  $OC$ . [2]
- (ii) Mark and **label** the point  $T$ . [1]
- (c) The police station,  $P$ , will be built so that it is equidistant from  $T$  and  $B$ .  
 It will be 3 km from  $O$  **and** North of  $AB$ .  
 Showing all your construction lines, find and **label** the point  $P$ . [3]
- (d) What will be the actual straight line distance between the town hall and the police station? [1]

5 The length,  $y$ , of a solid is inversely proportional to the square of its height,  $x$ .

(a) Write down a general equation for  $x$  and  $y$ .

Show that when  $x = 5$  and  $y = 4.8$  the equation becomes  $x^2y = 120$ .

(b) Find  $y$  when  $x = 2$ .

[1]

(c) Find  $x$  when  $y = 10$ .

[2]

(d) Find  $x$  when  $y = x$ .

[2]

(e) Describe exactly what happens to  $y$  when  $x$  is doubled.

[2]

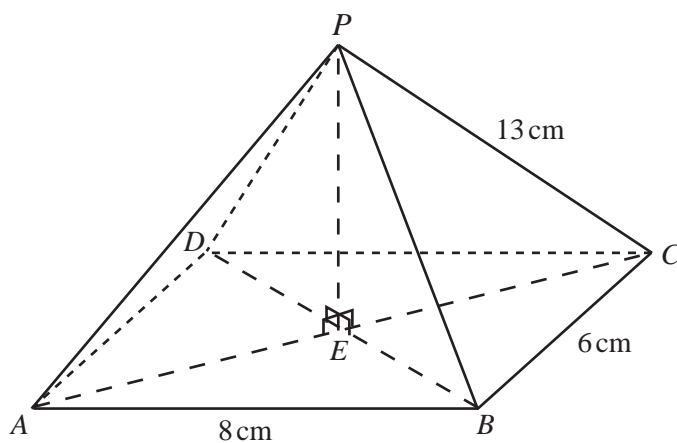
(f) Describe exactly what happens to  $x$  when  $y$  is decreased by 36%.

[2]

(g) Make  $x$  the subject of the formula  $x^2y = 120$ .

[2]

6



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The diagram shows a pyramid on a horizontal rectangular base  $ABCD$ .

The diagonals of  $ABCD$  meet at  $E$ .

$P$  is vertically above  $E$ .

$AB = 8$  cm,  $BC = 6$  cm and  $PC = 13$  cm.

(a) Calculate  $PE$ , the height of the pyramid.

[3]

(b) Calculate the volume of the pyramid.

[The volume of a pyramid is given by  $\frac{1}{3} \times \text{area of base} \times \text{height}$ .]

[2]

(c) Calculate angle  $PCA$ .

[2]

(d)  $M$  is the mid-point of  $AD$  and  $N$  is the mid-point of  $BC$ .

Calculate angle  $MPN$ .

[3]

(e) (i) Calculate angle  $PBC$ .

[2]

(ii)  $K$  lies on  $PB$  so that  $BK = 4$  cm.

Calculate the length of  $KC$ .

[3]

7 Transformation  $T$  is translation by the vector  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .

Transformation  $M$  is reflection in the line  $y = x$ .

(a) The point  $A$  has co-ordinates  $(2, 1)$ .

Find the co-ordinates of

(i)  $T(A)$ , [1]

(ii)  $MT(A)$ . [2]

(b) Find the 2 by 2 matrix  $\mathbf{M}$ , which represents the transformation  $M$ . [2]

(c) Show that, for any value of  $k$ , the point  $Q(k - 2, k - 3)$  maps onto a point on the line  $y = x$  following the transformation  $TM(Q)$ . [3]

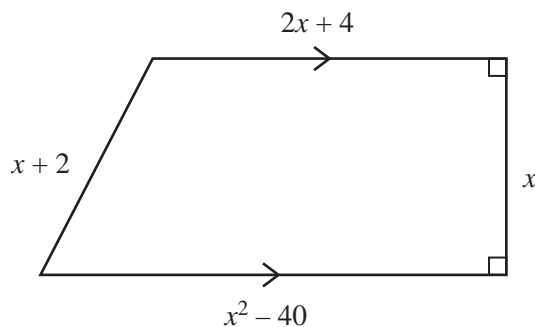
(d) Find  $\mathbf{M}^{-1}$ , the inverse of the matrix  $\mathbf{M}$ . [2]

(e)  $\mathbf{N}$  is the matrix such that  $\mathbf{N} + \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}$ .

(i) Write down the matrix  $\mathbf{N}$ . [2]

(ii) Describe completely the **single** transformation represented by  $\mathbf{N}$ . [3]

8 (a)



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The diagram shows a trapezium.

Two of its angles are  $90^\circ$ .

The lengths of the sides are given in terms of  $x$ .

The perimeter is 62 units.

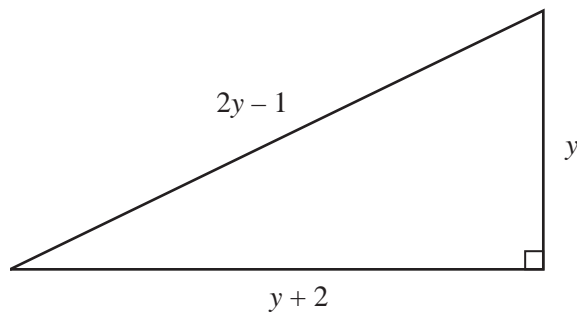
(i) Write down a quadratic equation in  $x$  to show this information. Simplify your equation. [2]

(ii) Solve your quadratic equation. [2]

(iii) Write down the only possible value of  $x$ . [1]

(iv) Calculate the area of the trapezium. [2]

(b)

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The diagram shows a right-angled triangle.  
The lengths of the sides are given in terms of  $y$ .

- (i) Show that  $2y^2 - 8y - 3 = 0$ . [3]  
 (ii) Solve the equation  $2y^2 - 8y - 3 = 0$ , giving your answers to 2 decimal places. [4]  
 (iii) Calculate the area of the triangle. [2]

- 9 (a) The numbers 0, 1, 1, 1, 2,  $k$ ,  $m$ , 6, 9, 9 are in order ( $k \neq m$ ).  
Their median is 2.5 and their mean is 3.6.

- (i) Write down the mode. [1]  
 (ii) Find the value of  $k$ . [1]  
 (iii) Find the value of  $m$ . [2]  
 (iv) Maria chooses a number at random from the list.  
The probability of choosing this number is  $\frac{1}{5}$ . Which number does she choose? [1]

- (b) 100 students are given a question to answer.  
The time taken ( $t$  seconds) by each student is recorded and the results are shown in the table.

$t$	$0 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 35$	$35 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 60$	$60 < t \leq 80$
Frequency	10	10	15	28	22	7	8

- (i) Calculate an estimate of the mean time taken. [4]  
 (ii) Two students are picked at random.  
What is the probability that they both took more than 50 seconds?  
Give your answer as a fraction in its lowest terms. [3]

**Answer part (c) on a sheet of graph paper.**

- (c) The data in **part (b)** is re-grouped to give the following table.

$t$	$0 < t \leq 30$	$30 < t \leq 60$	$60 < t \leq 80$
Frequency	$p$	$q$	8

- (i) Write down the values of  $p$  and  $q$ . [2]  
 (ii) Draw an accurate histogram to show these results.  
Use a scale of 1 cm to represent 5 seconds on the horizontal time axis.  
Use a scale of 1 cm to 0.2 units of frequency density (so that 1 cm<sup>2</sup> on your histogram represents 1 student). [4]

